Further Pure Core – Series

Mostly taken from OCR past papers.

1. Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n}(6r^2 + 2r + 1) = n(2n^2 + 4n + 3).$$

2. Use the standard results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n}(8r^3 - 6r^2 + 2r) = 2n^3(n + 1).$$

3. Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n}(r^3 + r^2) = \frac{n}{12}(n + 1)(n + 2)(3n + 1).$$

4. Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$ to evaluate

$$\sum_{r=1}^{n} r(r - 1)(r + 1),$$

expressing your answer in a fully factorised form.

5. Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n}(3r^2 - 3r + 1) = n^3.$$

6. Find $\sum_{r=1}^{n} r^2(r - 1)$, expressing your answer in a fully factorised form.

7. Evaluate $\sum_{r=101}^{250} r^3$.

8. Given that $\sum_{r=1}^{n}(ar^3 + br) \equiv n(n - 1)(n + 1)(n + 2)$, find the values of the constants a and b.

9. (a) From the facts

$$1 = 0 + 1$$
$$2 + 3 + 4 = 1 + 8$$
$$5 + 6 + 7 + 8 + 9 = 8 + 27$$
$$10 + 11 + 12 + 13 + 14 + 15 + 16 = 27 + 64$$

guess a general law. Prove it.

(b) Hence prove that

$$1^3 + 2^3 + 3^3 + \cdots + N^3 = \frac{1}{4}N^2(N+1)^2$$

for every positive integer N.

[You may assume that $1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n+1)$.] [STEP]